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Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2017

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

* J M A T H E 2 *

This question paper consists of 14 pages, including 1 page information sheet, and a SPECIAL ANSWER BOOK.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary round off your answers to TWO decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The percentages obtained by learners in their first Mathematics test is shown in the table below.

Percentages	Frequency	Cumulative Frequency
$30 \leq x < 40$	1	
$40 \leq x < 50$	2	
$50 \leq x < 60$	9	
$60 \leq x < 70$	12	
$70 \leq x < 80$	11	
$80 \leq x < 90$	9	
$90 \leq x < 100$	6	

1.1 Complete the cumulative frequency column in the table given in the ANSWER BOOK. (3)

1.2 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (4)

1.3 Estimate how many learners obtained 75% or less for the test. Indicate this by means of B on your graph. (2)

[9]

QUESTION 2

The water consumption (in kilolitres) of 15 households is as follows:

12,4	20,0	34,5	40,1	18,9
19,7	34,9	15,1	23,8	23,7
31,1	20,9	19,7	36,5	33,6

2.1 List the five number summary for the data. (4)

2.2 Draw a box-whisker diagram to represent the data. (3)

2.3 Comment on the skewedness of the data represented in QUESTION 2.2. (1)

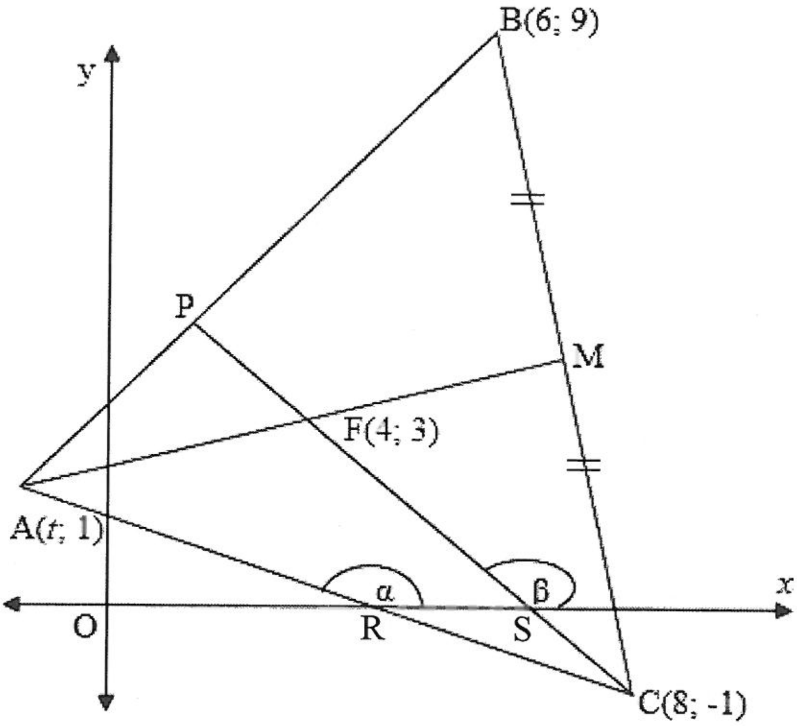
2.4 Determine the standard deviation of the data. (2)

2.5 Use the standard deviation to comment on the spread of the data. (1)

[11]

QUESTION 3

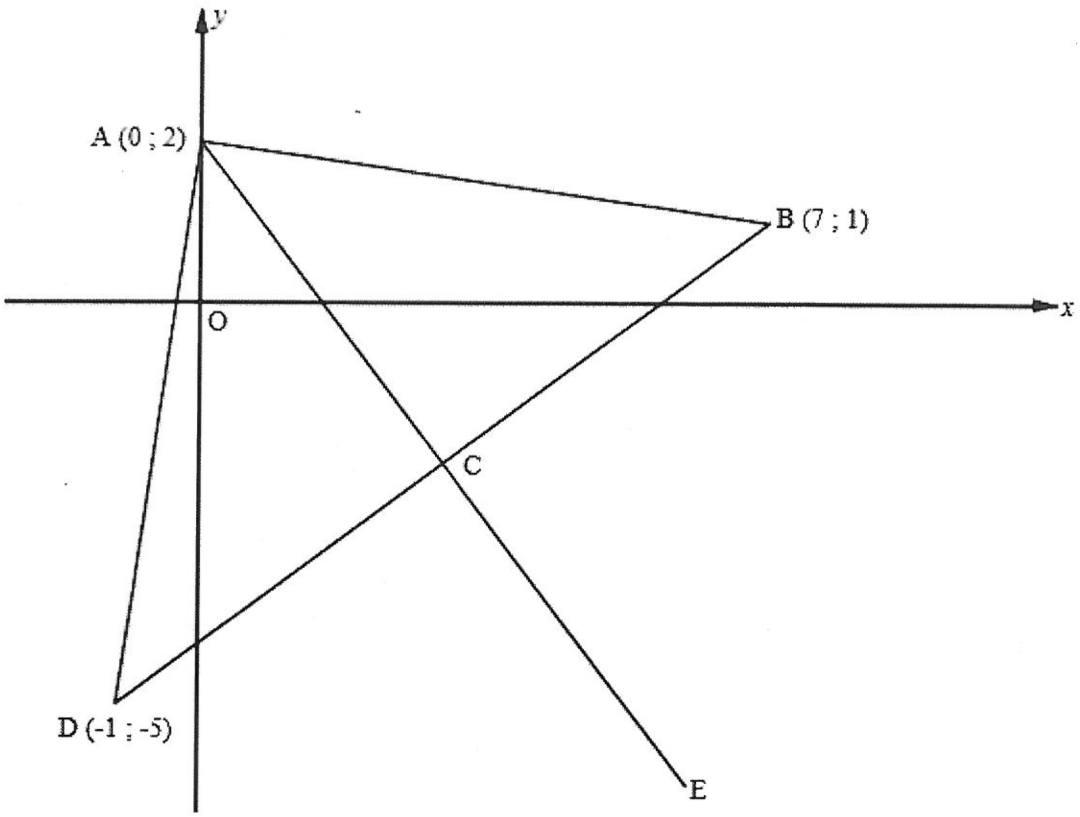
In the diagram, A (t ; 1), B (6 ; 9) and C (8 ; -1) are points in a Cartesian plane.
M is the midpoint of BC. P is a point on AB. CP intersects AM at F (4 ; 3). R is the x-intercept of line AC and S is the x-intercept of line PC.



3.1	Calculate the coordinates of M.	(2)
3.2	Determine the equation of the median AM.	(4)
3.3	Calculate the value of t.	(2)
3.4	Calculate the gradient of PC.	(2)
3.5	Determine the size of β.	(2)
3.6	Calculate the size of $A\hat{C}P$.	(4)
		[16]

QUESTION 4

Quadrilateral ABED, with vertices A (0 ; 2), B (7; 1), D (-1 ; -5) and E is given below. Diagonals AE and BD intersect at C.



4.1 Calculate the coordinates of C, the midpoint of BD. (2)

4.2 Show that CA = CB if the coordinates of C are (3 ; -2). (3)

4.3 Why is $D\hat{A}B = 90°$? (5)

4.4 Hence, write the equation of the circle with centre C which is passing through A, B, E and D. (2)

4.5 Calculate the gradient of BC, the radius of the circle. (2)

4.6 Determine the equation of the tangent to the circle at B in the form $y = \ldots$ (3)

4.7 Explain why ABED is a rectangle. (3)

[20]

QUESTION 5

5.1 If $\sin 58° = k$, determine, **without the use of a calculator:**

5.1.1 $\sin 238°$ (2)

5.1.2 $\cos 58°$ (2)

5.2 Simplify, **without the use of a calculator**:

$$\frac{\tan 150°.\sin 300°.\sin 10°}{\cos 225°.\sin 135°.\cos 80°}$$ (7)

5.3 Given $\cos(\alpha + \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Use the formula for $\cos(\alpha + \beta)$ to derive a formula for $\sin(\alpha + \beta)$. (4)

5.4 Prove the identity: $$\frac{\cos 2x + 1}{\sin 2x.\tan x} = \frac{1}{tan^2 x}$$ (4)

5.5 5.5.1 Show that $\tan x = 2\sin x$ can be written as $\sin x = 0$ or $\cos x = ½$. (3)

5.5.2 Hence, write down the general solution of the equation

$$\tan x = 2\sin x$$ (4)

[26]

QUESTION 6

Given $f(x) = \tan x$ and $g(x) = \sin(x + 45°)$

6.1 Draw the graphs of *f(x)* and g(*x)* on the same set of axes for $x \in [-90°; 180°]$, on the grid provided in the ANSWER BOOK. (6)

6.2 Use your graphs to determine the value(s) of *x* in the interval $x \in [-90°; 90°]$ for which:

6.2.1 $g(x) - f(x) = 1$ (2)

6.2.2 $g(x) \geq f(x)$ (2)

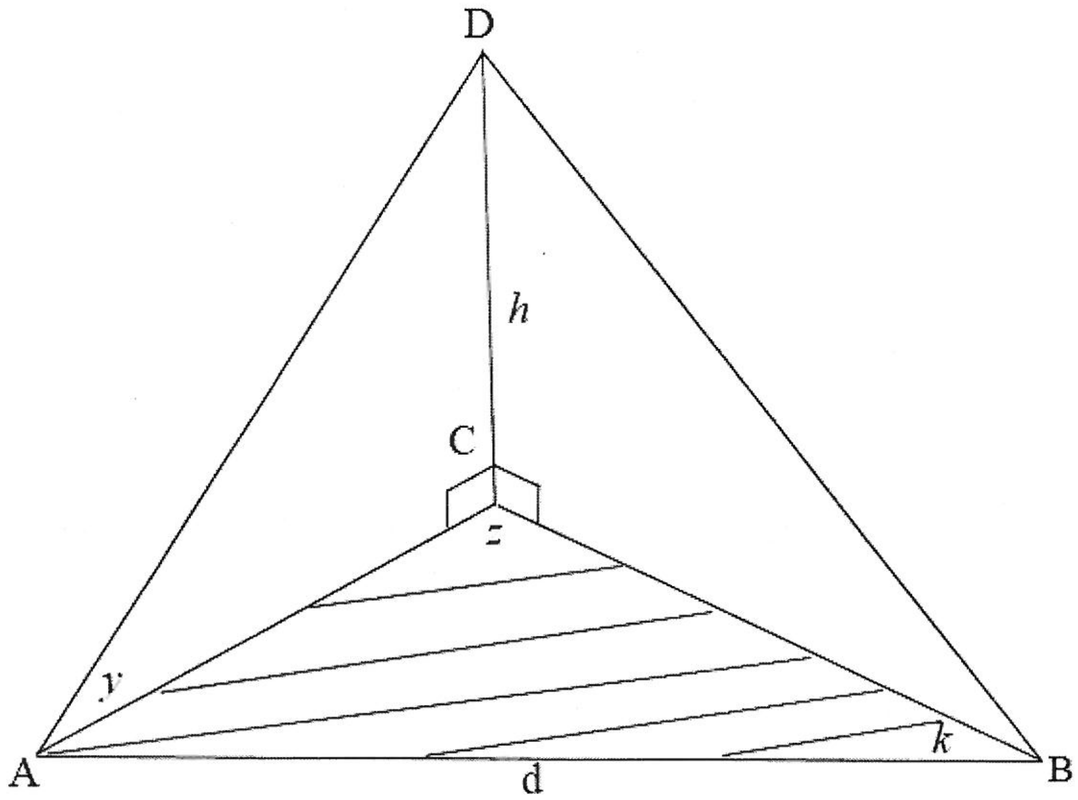
6.3 State the period of $y = f(2x)$. (1)

[11]

QUESTION 7

To find the height h of a tree CD, the end of the shadow was marked at points A and B in the same horizontal plane as its stem C at different times of the day. The shadow of the tree rotated $z°$ between the times of observation, i.e. $A\hat{C}B = z°$.

AB = d metres, $A\hat{B}C = k°$ and the angle of elevation of the sun at A was $y°$.



7.1 Find the length of AC in terms of z, k and d. (2)

7.2 Find the length of AC in terms of y and h. (2)

7.3 Hence show that $h = \dfrac{d \sin k . \tan y}{\sin z}$. (1)

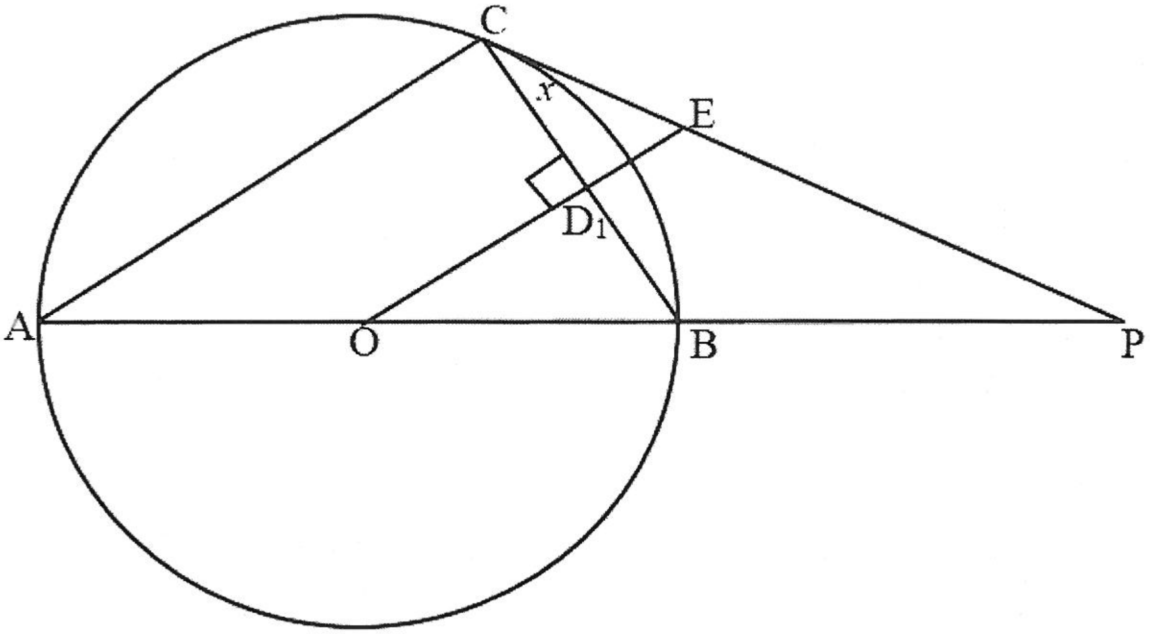
7.4 Calculate the length of h if $z = 125°, d = 80m, k = 38°$ and $y = 40°$. (2)

[7]

Give reasons for ALL statements in QUESTION 8, 9, 10 AND 11.

QUESTION 8

In the figure, AB is a diameter of the circle with centre O. AB is produced to P. PC is a tangent to the circle at C and line ODE perpendicular to BC intersects BC at D and PC at E.



8.1 Give a reason why CD = DB. (1)

8.2 Show that AC || OE. (3)

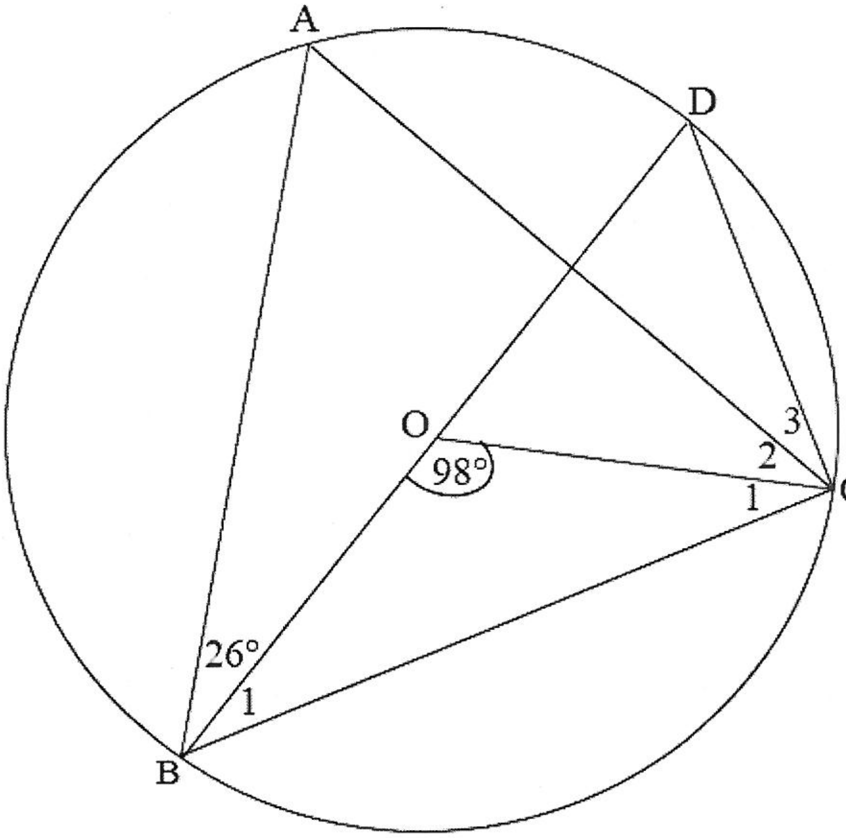
8.3 If $B\hat{C}P = x$, name two other angles equal to x. (4)

8.4 Prove that OBEC is a cyclic quadrilateral. (2)

[10]

QUESTION 9

In the diagram, BD is the diameter of the circle ABCD with centre O. $A\hat{B}D = 26°$ and $B\hat{O}C = 98°$.



Calculate:

9.1 $\hat{A}$ (2)

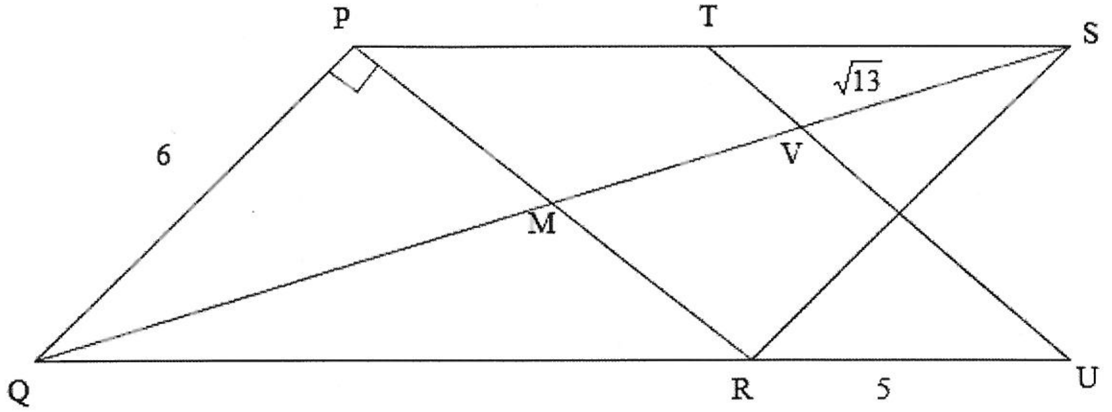
9.2 $\hat{B}_1$ (3)

9.3 $\hat{C}_2$ (3)

[8]

QUESTION 10

In the diagram below PQRS is a parallelogram, with the diagonals intersecting at M.
$Q\hat{P}R = 90^0$. QR is produced to U. T is a point on PS. TU intersects QS at V.
$PQ = 6, \quad PR = 8, \quad RU = 5$ and $VS = \sqrt{13}$



10.1 Determine with reasons the following ratios in simplified form:

10.1.1 $\frac{UR}{RQ}$ (3)

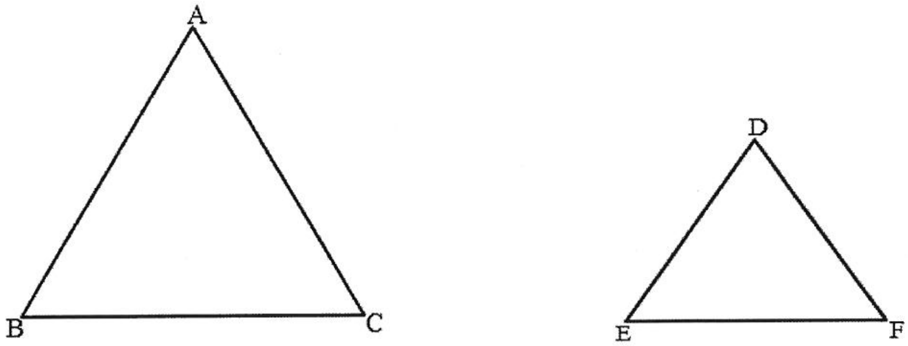
10.1.2 $\frac{VM}{MQ}$ (4)

10.2 Hence, prove that $MR \parallel VU$ (2)

[9]

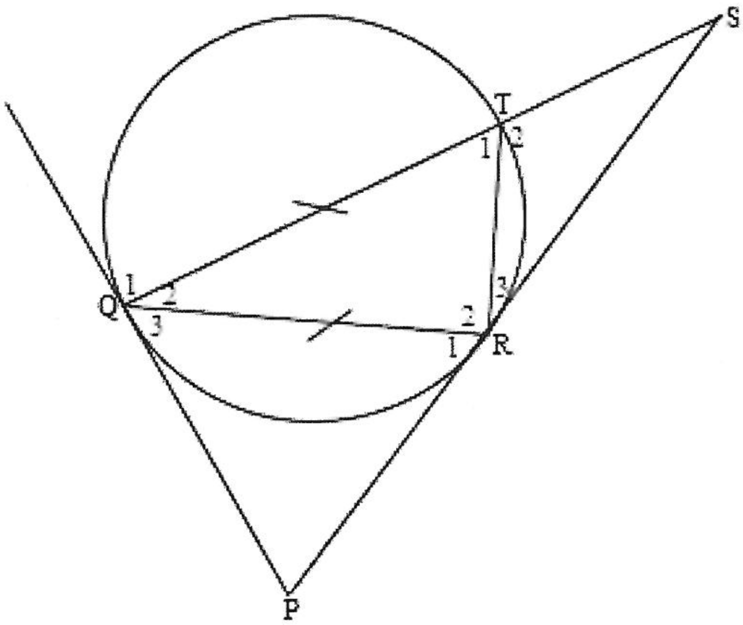
QUESTION 11

11.1 In ΔABC and ΔDEF, $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$, respectively. Prove that $\frac{AB}{DE} = \frac{AC}{DF}$.



(7)

11.2 Tangents PQ and PR touch the circle at Q and R respectively. T is a point on the circle such that QT = QR. QT and PR are produced and they meet at S. $\hat{Q}_1 = x$.



11.2.1 Name THREE other angles equal to x. (3)

11.2.2 Determine, in terms of x, the size of $\hat{Q}_2$. (2)

11.2.3 Hence show that TR || QP. (3)

11.2.4 Prove that $\Delta STR \parallel\!| \Delta SRQ$. (3)

11.2.5 Hence show that $RS^2 = ST \times SQ$. (2)

11.2.6 If it is further given that $QT : TS = 3 : 2$, show that $\frac{SP}{PQ} = \frac{5}{3}$. (3)

[23]

TOTAL: 150

INFORMATION SHEET MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad A = P(1 - ni)$ $\quad A = P(1 - i)^n$ $\quad A = P(1 + i)^n$

$\sum_{i=1}^{n} 1 = n$ $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\quad T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$